STUDY OF GENERALIZED FRACTIONAL q-INTEGRAL OPERATORS ASSOCIATED WITH GENERALIZED BASIC HYPER-GEOMETRIC SERIES

SYNOPSIS

SUBMITTED TO THE

MAHARSHI DAYANAND SARASWATI UNIVERSITY, AJMER

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE AWARD OF THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

MATHEMATICS

By

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OUTLINE OF PROPOSED RESEARCH WORK

(SYNOPSIS)

1. **Name of Scholar (In English):-** LOKESH JASORIA

   (In Hindi):- लोकेश जसोरिया

2. **Title of the Research Work: (In English):**
   STUDY OF GENERALIZED FRACTIONAL q-INTEGRAL OPERATORS ASSOCIATED WITH GENERALIZED BASIC HYPER-GEOMETRIC SERIES.

   (In Hindi) - व्यापकीकृत भिन्नात्मक q-समाकल संकारकों से संबद्ध व्यापकीकृत मौलिक उच्च ज्यामितीय श्रेणीयों का अध्ययन।

3. **Location –**

   a. **Institution/ Department where the work is to be done:-**

   DEPARTMENT OF MATHEMATICS
   GOVERNMENT COLLEGE OF AJMER,
   RAJASTHAN-INDIA

   b. **Geographical area of investigation: - NIL**

4. IMPORTANCE OF PROPOSED INVESTIGATION, OBJECTIVES.

The concept of fractional calculus is believed to have stemmed from a question raised by L'Hospital to Leibniz in 1695. It has gained considerable attention during last four decades due to its distinguished applications in numerous diverse fields of science and engineering especially in the areas like ordinary calculus, optimal control problems, solutions of the q-difference (differential) equations, q-transform analysis and many more. The study of basic hyper geometric series was initiated in early 20^{th} century. A detail account of which can be seen in the books by Slater [1966.], Exton [1983], Gasper [1990]. Basic hyper-geometric series are called q-analogues (basic analogues or q-extensions) of hyper-geometric series because an ${}_rF_s$ series can be obtained as the $q \to 1$ limit case of an ${}_r\varphi_s$ series. Gasper and Rahman [1990] have derived a q-analogue of it, called the q-binomial theorem, and used it to derive Heine's q-analogues of Euler's transformation formulas, Jacobi's triple product identity, and summation formulas that are q-analogues of those for hyper-geometric series due to Chu and Vandermonde, Gauss, Kummer, Saalschutz, and to Karlsson and Minton. They have also introduced q-analogues of the exponential, gamma and beta functions, as well as the concept of a q-integral that allows us to give a q-analogue of Euler's integral representation of a hyper-geometric function. Al-Salam [1952-53] introduced the concept of fractional q-calculus, starting from the q-analogue of Cauchy's formula. Agarwal [1969] studied certain fractional q-integral operators and q-derivatives, where he proved the semigroup properties for left and right Riemann-Liouville type fractional integral operators. Further, Iso-gawa et al. [2007] studied some basic properties of fractional q-derivatives. Rajkovic et al. [2007] generalized the notion of the left fractional q-integral operators and fractional q-derivatives by introducing variable lower limit and proved the semi-group properties. Garg et al. [2011] introduced q-analogues of hyper-

Bessel type Kober fractional derivatives. Further, Saxena et al. [2005] and Yadav et al. [2008] have obtained images of various q-special functions under fractional q-calculus operators.

Purohit and Yadav [2006] have defined q-extensions of the Saigo's[1978] fractional integral operators .Studies on q-difference equations appeared already at the beginning of the last century in intensive works especially by Jackson [1910],Carmichael [1912], Mason [1915], Adams [1929], Trjitzinsky [1933] and other authors such us Poincare, Picard and Ramanujan.

Since 1980's an intensive and somewhat surprising interest in the subject reappeared in many areas of mathematics like applications including mainly new difference calculus and orthogonal polynomials, q-combinatorics, q-arithmetic's , q- integrable systems and variational q-calculus.

Keeping in view the wide applicability of this the objectives of proposed investigation will be:-

- To study fractional q-integrals involving generalized basic hyper geometric series.
- To obtain the q-extension of Generalized multivariable fractional integral operators and Bounded integral operators.
- To extend known results of Erdéyli type integrals, Weyl type integrals, q-Riemann–Liouville fractional integral operator, Saigo's fractional integral operators, and their applications.

5. REVIEW OF WORK ALREADY DONE ON THE TOPIC:

The q-Riemann–Liouville fractional integral operator introduced by Al-Salam (1966-1967) and by Agarwal (1969). An excellent survey of the work done on operators of fractional integration and their applications has been made in the highly useful work of Srivastava and Saxena [2001]. Further, theory and applications of fractional differential equations have been recently studied by

Kilbas et al. [2006]. Generalized multivariable fractional integral operators have been studied from time to time by several authors including Srivastava et al. [1995], Gupta et al. [1995] and Saxena et al. [1994].

Michael Schlosser (http://random.mat.univie.ac.at/people/mschloss) has derived summation formulas for a specific kind of multidimensional basic hyper-geometric series associated to root systems of classical type. He proceeded by combining the classical (one-dimensional) summation formulas with certain determinant evaluations. He also derived multidimensional extensions, associated to the classical root system of type A_r, B_r, Cr, and D_r, respectively of Chu's bilateral transformation formula for basic hyper-geometric series of Gasper-Karlsson-Minton type.

Tim Huber (2007) has derived product representation for generalization of the Rogers-Ramanujan series. Special cases of the results were first stated by Ramanujan and presented in the "Lost Notebook" and proved by Andrews. The analysis used in this paper is based upon the work of Andrews and the broad contributions made by Ismail and Hayman. Each series considered is related to an extension of the Rogers–Ramanujan's continued fraction and corresponds to an orthogonal polynomial sequence generalizing classical orthogonal sequences. Using Ramanujan's differential equations for Eisenstein series and corresponding analogues derived by Ramamani, the coefficients in the series representations of each zero are expressed in terms of certain Eisenstein series.

Yadav, Purohit and Kalla (2008) have studied Fractional q-integral operators of generalized Weyl type; involving generalized basic hyper-geometric functions and a basic analogue of Fox's H- function have also been investigated. A number of integrals involving various q-functions have been evaluated as applications of the main results.

Gupta and Agrawal (2009) have established a general theorem that gives the

image of a modified H-transform in the fractional integral operator involving the multivariable H-function. Next, they deduce two important corollaries involving Wright's generalized Bessel function, Mittag-Leffler function, Appell function F_1 and the product of Whittaker functions which are also quite general in nature and of interest to mathematician. Several other new and known results can also be obtained from their main theorem.

Thomas Ernst (2010) has obtained four q-analogues of general reduction formulas from Buschman and Srivastava (1982) together with some special case, e.g. q-analogues of reduction formulas for Appell and Kampe de Feriet function. A proper q-analogue of the notation Δ (l; λ) by Mac Robert (1959), Meijer and Srivastava (1979) have been given, and the definition of q-hyper-geometric series have also been generalized accordingly.

Yadav and Purohit (2010) have introduced two generalized operators of fractional q-integration, which may be regarded as extensions of Riemann-Liouville, Weyl and Kober fractional q-integral operators. Certain interesting connection theorems involving these operators and q-Mellin transform have also been discussed.

6. RESEARCH GAPS IDENTIFIED IN THE PROPOSED FIELD OF INVESTIGATION:

Recently fractional q-integral operators of Erdely, Kober and Weyl type have been studied. Here our attempt will be to obtain the fractional q-integral of Saigo and other integral operators to establish some new results involving these fractional q-integrals and generalized basic hyper-geometric series.

7. RESEARCH METHODOLOGY:

The following method will be used for the present investigation. Basic hyper-geometric series, Erdéyli type integrals, q-Riemann–Liouville fractional integral operator, Saigo's fractional integral operators, Generalized

multivariable fractional integral operators, q-Bessel operator, q-Mittag-Leffler function, Bailey's transform, Rogers-Ramanujan identities. Library and internet facility will be the main source to collect the material for present investigation.

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Government College Ajmer.